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# Indian Maritime University

**(A Central University, Govt of India)**

**Sep/Oct'25 SE**

**Programme Name: B Tech (ME)**

**Semester: First**

**Subject Code: UG11T5102**

**Subject Name: Engineering Mathematics 1**

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| Date: 02.09.2025 | Max Marks: 70 |
| Duration: 03 Hrs | Pass Marks: 35 |

## Section A (10X1=10 Marks)

**Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.**

1. The nullity of an invertible matrix is

   a) 1 b)0 c) 2 d)3

2. Consider the following matrix A. Which is the correct statement.

$$A=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

   a) The matrix A is not a diagonal matrix.
   b) The eigen values of matrix are 1, 2 and 4
   c) The eigen values of matrix are 1 and 2
   d) None of the other options

3. The complementary function of $(D^3+D^2-D-1)y=0$ is

   a) $c_1e^x-(c_2+c_3x)e^{-x}$ b) $c_1e^{-x}+(c_2+c_3x)e^x$

   c) $c_1e^x+(c_2+c_3x)e^{-x}$ d) $c_1e^{-x}+(c_2+c_3x)e^x$

4. If the function(x)= $x^2$-4x+3 on the interval [1,3] satisfies the Rolle's theorem

then the value of c is ________

a) 1 b) 2 c)3 d) -2

5. The number of arbitrary constants in the particular solution of a differential equation of third order is:

a) 3 b) 2 c) 1 d) 0

6. Expansion of cos x in ascending powers of x is ________

a) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

b) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}$

c) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots$

d) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}\cdots$

7. $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dxdy=$

a) $\frac{3}{35}$ b) $\frac{7}{60}$ c) $\frac{4}{49}$ d) $\frac{2}{15}$

8. The value of the integral $\int_{-1}^{1}\int_{1}^{2}\int_{0}^{1} dxdydz$ is

a) 2 b) 0 c) 1 d) 4

9. The value of the line integral $\int_c(2xy^2dx+2x^2ydy+dz)$ along a path joining the origin and the point (1,1,1) is

a) 0 b) 2 c) 4 d) 6

10. Find a unit vector normal to the surface $xy^3z^2=4$ at the point (-1,-1,2).

a) $\frac{\vec{i}+3\vec{j}+\vec{K}}{\sqrt{11}}$ b) $\frac{\vec{i}-3\vec{j}-\vec{K}}{\sqrt{11}}$ c) $\frac{\vec{i}+3\vec{j}-\vec{K}}{-\sqrt{11}}$ d) $\frac{\vec{i}+3\vec{j}-\vec{K}}{\sqrt{11}}$

## Section B

**Five Questions of 02 Marks each** **(5X2=10 Marks)**

11. Verify the rank and nullity theorem for the matrix A=$\begin{bmatrix}1 & 3 & 4 & 3\\ 3 & 9 & 12 & 9\\ 1 & 3 & 4 & 1\end{bmatrix}$

12. Find nth order derivative of $y=e^{3x}\cos^2 x$

13. If u=xy-yz-zx, v=$x^2+y^2+z^2$, $w=x+y-z$, determine whether they are functionally related or not, if so find the relationship between them.

14. Evaluate $\int_0^a \int_0^b \int_0^c x^2y^2z^2 \, dx\, dy\, dz$

15. Find div $\bar{F}$ and curl $\bar{F}$ where $\bar{F}$ = $(3x^2 - 3yz)$ i + $(3y^2 - 3zx)$ j + $(3z^2 - 3xy)$ k

## <u>Section C</u>

**Seven Questions of 10 Marks each of which any 05 questions to be answered. (5X10=50 Marks)**

16. a) Test for consistency (5 marks)

$x + y + z = 6$ ; $x + 2y + 3z = 14$; $x + 4y + 7z = 30$ and solve them.

16. b) Find the characteristic roots of the matrix A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and

(i) Verify Caley-Hamilton theorem for this matrix.

(ii) Find $A^{-1}$ (3+2=5 marks)

17. a) Find $n^{th}$ derivative of $y = \left[\frac{1}{(x+2)(x+1)(x-1)}\right]$ . (5 marks)

17. b) Solve $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ (5 marks)

18. a) If u= log (tan x+ tan y+ tan z) show that (5 marks)

Sin 2x $u_x$ + Sin 2y $u_y$ + Sin 2z $u_z$ =2

18. b) By Euler's theorem for homogeneous function (5 marks)

u= $\frac{(x^2+y^2)^m}{2m\,(2m-1)} + x\varphi\left(\frac{y}{x}\right) + \Psi\left(\frac{y}{x}\right)$,

Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (x^2+y^2)^m$

19. a) Change the order of integration and hence evaluate the integral

$$\int_0^1 \int_x^{2-x} \frac{x}{y}\, dxdy$$ (5 marks)

19. b) Find the area between the parabolas $y^2$ =4ax and $x^2$ =4ay. (5 marks)

20. a) If $y=(x^2-1)^n$ , then show that

$(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$ (5 marks)

20. b) Using Lagranges's method of undetermined multipliers find the maximum and minimum distances of the point (3,4,12) from the sphere

$x^2 + y^2 + z^2 = 4.$ (5 marks)

21. a) The upward speed $v(t)$ of a rocket at time t is approximated by

$v(t) = at^2 + bt + c$, $0 \leq t \leq 100$ where a, b, and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second. Find the speed at time t = 15 seconds. Solve the problem using Gaussian Elimination. (6 marks)

21. b) Find the eigenvalues and eigenvectors of A= $\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ (4 Marks)

22. a) Prove that $\overrightarrow{A} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational.

Find the scalar function f(x,y,z) such that $\overrightarrow{A} = \nabla f$ (5 marks)

22. b) Examine whether the given equation is exact or not and hence solve it

$(x^2 + y^2 + x)dx + xydy = 0$ (5 marks)