INDIAN MARITIME UNIVERSITY

(A Central University, Govt. of India)

END SEMESTER EXAMINATIONS- JUNE-JULY 2019

B. SC. (NAUTCAL SCICENCE)

SEMESTER III

APPLIED MATHEMATICS III

(UG21T2302)

Date: 27.06.2019 **Max.Marks: 70**

Time: 3 Hours **Pass Marks: 35**

Note: Answer any Seven Questions out of Nine Questions.

Q.1 (a) Find the Laplace transform of $(sint + cost)^2$ (5 Marks)

(b) i)Find the Laplace transform of $\frac{1-e^t}{t}$

ii) Find the Laplace transform of $e^{2t}cos^2t$ (5 Marks)

Q.2 (a) Find the inverse Laplace transform of $\frac{s+2}{s^2-4s+13}$ (5 Marks)

(b) Find the inverse Laplace transform of $\frac{1}{(s^2+a^2)^2}$ (5 Marks)

Q.3 (a) Using convolution theorem evaluate $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ (5 Marks)

(b) Evaluate $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$ (5 Marks)

Q.4 (a) Solve by the method of transforms, the equation

$y''' + 2y' - y' - 2y = 0 \; given \; y(0) = y'(0) \; and \; y''(0) = 6$ (5 Marks)

(b) Find the inverse Laplace transform of $\frac{s^2 - 3s + 4}{s^3}$ (5 Marks)

Q.5 Solve the simultaneous equations by using Laplace transforms:

$\frac{dx}{dt} + 5x - 2y = t, \quad \frac{dy}{dt} + 2x + y = 0 \quad being \; given \;\; x = y = 0 \; when \; t = 0$ (10 Marks)

Q.6 (a) Prove that $J_1''(x) = -J_1(x) + \frac{1}{x} J_2(x)$ (5 Marks)

(b) Prove that $\int J_3(x)dx = c - J_2(x) - \frac{2}{x} J_1(x)$ (5 Marks)

Q.7 (a) Prove that $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$ (5 Marks)

(b) Show that $\int_{-1}^{1} x^2 P_{n-1} P_{n+1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ (5 Marks)

Q.8 Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials (10 Marks)

Q.9 Show that for any function f(x) for which the nth derivative is continuous,

$\int_{-1}^{1} f(x) P_n(x) dx = \frac{1}{2^n \, n!} \int_{-1}^{1} (1 - x^2)^n f^n(x) dx$ (10 Marks)