Indian Maritime University

(A Central University, Govt of India)

End Semester Examinations – June 2023

Programme Name: B Tech (ME)

Semester: I

Subject Code: UG11T4101

Subject Name: Mathematics I

Date: 13.06.2023	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

General Instructions

(i) All Sections (A, B & C) are to be attempted.
(ii) Options, if any, are specified in respective section.
(iii) Scientific calculator is permitted.

Section A

MCQs. Choose the correct answer as applicable. [10X1=10]

1. Expansion of cosh x in ascending powers of x is

A. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

B. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$

C. $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \ldots$

D. $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \ldots$

2. If $x^3 + y^3 = 6xy$ then $\frac{dy}{dx}$ is

A. $\frac{2y-x^2}{y^2-2x}$

B. $\frac{x^2-2y}{y^2+2x}$

C. $\frac{-x^2}{y^2+2}$

D. $\frac{x^2}{y^2-2}$

3. $\lim_{x \to 0} \frac{\sin x}{x}$ is equal to

A. 2

B. 1/2
C. 1
D. none of above

4. For n x n homogeneous system of linear equations AX=0 is given . If the rank of A is r <n, then system has

A. n-r independent solutions
B. r independent solutions
C. no solutions
D. n-2r independent solutions

5. The Eigen values of the matrix A = $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

A. λ=6,1
B. λ=-4, -5
C. λ=4,6
D. λ=-4, -6

6. Divergence of vector field $x^2z\hat{i} + xy\,\hat{j} - yz^2\hat{k}$ at (1, -1, 1) is

A. 0
B. 3
C. 5
D. 6

7.The value of β $\left(\frac{1}{2},\frac{1}{2}\right)$ =________

A. 0
B. 1
C. ½
D. π

8. If is a homogeneous function of degree in 'n' in x and y, then

A. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

B. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = nu$

C. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n+1)u$

D. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n^2u$

9. The value of the integral $\int_{-1}^{1}\int_{-1}^{1}(2x+y)\,dx\,dy$ is

A. -2
B. 0
C. 1
D. 3/2

10. Value of $\int_0^a \int_0^b \int_0^c x^2y^2z^2 \;\; dx\, dy\, dz$ is _____

A. $\frac{abc}{3}$
B. $\frac{a^2b^2c^2}{27}$
C. $\frac{a^3b^3c^3}{27}$
D. $\frac{a^3b^3c^3}{9}$

Section B

Answer the following [5x2 = 10 Marks]

11. Find n^{th} derivative of log $(4x^2 - 1)$.

12. Evaluate $\int_0^{\infty} \frac{x^4}{4^x}\, dx$ by using Gamma function.

13. Find the degree of the homogeneous function $u = \dfrac{3x^2 + 4y^2}{3x + 4y}$.

14. Find the characteristic roots of the matrix A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find A^{-1}.

15. If x = r cos θ , y= r sin θ show that $\left(\frac{\partial r}{\partial x}\right) = \left(\frac{\partial x}{\partial r}\right)$.

Section C

Answer any 5 out of 7 questions. [5x10 = 50 Marks]

16. a) If y = $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ Prove that $(1-x^2)\, y_{n+1} - (2n+1)\, x y_n - n^2 y_{n-1} = 0$ (05)

b) If v= e^{2x} (y cos2y + x sin2y) then show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ (05)

17. a) Find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ if $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ (05)

b) Show that the rectangular solid of volume V= 8xyz that can be inscribed in a sphere $x^2+y^2+z^2=r^2$ is a cube. (05)

18. a) Evaluate $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx$, using Beta function. (05)

b) Calculate $\iint r^3 dr\, d\theta$ over the are included between the circles

r = 2 sin θ and r = 4 sin θ (05)

19. a) Change the order of integration in $I=\int_0^4\int_{x^2/4}^{2\sqrt{x}} dy\, dx$ and hence evaluate. (05)

b) Evaluate $\iiint_V \frac{dx\, dy\, dz}{(x+y+z+1)^3}$ where V is the region of space bounded by x=0, y=0,z=0 and x+y+z=1. (05)

20. a) Test the following system of equations for consistency and if

consistent solve

x+2y+z=3: 2x+3y+2z=5 ; 3x-5y+5z=2 ; 3x+9y-z=4 (05)

b) Find the Eigen values and Eigen vectors of the matrix A= $\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$ (05)

21. a) A particle moves along a curve x= 2 sin 3t, y= 2 cos 3t , z= 8t at any time t=0 where t is the time. Find the magnitude of its velocity and acceleration. Find unit tangent vector to the curve. (05)

b) Find the directional derivative of f= x^2-y^2 -2z^2 at point P (2, -1, 3) in the direction of line PQ where Q(5,6,4) . (05)

22. a) Prove that $\bar{F}=(6xy+z^3)\bar{\imath}+(3x^2-z)\bar{\jmath}+(3xz^2-y)\bar{k}$ is irrotational . Find a scalar function $\emptyset$ (x,y,z)such that $\bar{F}=\nabla\emptyset$. (05)

b) Given $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & -1 & -2 & d^2 \\ -1 & -7 & -11 & d \end{bmatrix}$ the augmented matrix of the system of equations,

where 'd' is the real number. Determine the all values of 'd' so that the corresponding system is consistent. (05)