INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

END SEMESTER EXAMINATION-DECEMBER 2019

B.Sc(Nautical Science)

Semester – I

Nautical Mathematics

(UG21T4102)

Date: 12.12.2019	Max Marks: 70
Time: 3 Hrs	Pass Marks : 35

Note: Part A is compulsory.

Answer any 6 from remaining 8 questions of Part B.

PART A

(5 x 2 = 10 marks)

1. a. Find the n^{th} derivative of $x^2 \log 3x$.

   b. Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2}$

   c. Separate into real and imaginary part of $\exp\left(5 + \frac{i\pi}{2}\right)$.

   d. Express $\int_0^{\pi/2} \sqrt{\cot\theta}\ d\theta$ in terms of Gamma function.

   e. Verify Rollis theorem for $f(x) = (x+2)^3(x-3)^4$ in (-2, 3)

PART B

2. a. Change the order of integration and evaluate

$$\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$$

   b. Find the area lying between the parabola $y = x^2$ and the line $x + y = 0$ by double integration.

(5+5 marks)

3. a. Evaluate

$$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$$

   b. Evaluate $\int_0^\infty e^{-ax} x^{m-1} \sin bx\,dx$ in terms of Gamma functions.

(5+5 marks)

4. a. In spherical triangle PQR angle $P = 53°5'$ sides $PQ = 70°20'$ and $PR = 110°\,14'$. Calculate angle Q.

   b. In spherical triangle RST side $t = 80°\,32'$, side $r = 60°\,40'$ and angle $T = 90°$. Calculate side s.

(5+5 marks)

5. a. In spherical triangle LMN angles $N = 81° 50'$and $L = 119° 7'$. Side $m = 90°$. Calculate angle M.

b. In spherical triangle PQR angles $Q = 74° 52' 18''$and $R = 71° 20'$. Side$p = 49° 8'$. Calculate side r.

(5+5 marks)

6. a. If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

b. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

(5+5 marks)

7. a. Find the minimum value of $x^2 + y^2 + z^2$ given $ax + by + cz = p$.

b. If $Z = f(x,y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

(5+5 marks)

8. a. If $p = CiS\,\theta$ and $q = CiS\,\emptyset$ show that $\frac{p-q}{p+q} = i\tan\left(\frac{\theta-\emptyset}{2}\right)$.

b. Find all the values of $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{3/4}$. Also show that the continued product of these valuesare 1.

(5+5 marks)

9. a. Expand $\cos^8\theta$ in a series of cosine multiples ofθ .

b. Show that $\sin h^{-1}(\tan\theta) = \log\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$

(5+5 marks)

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