Indian Maritime University

(A Central University, Govt of India)

Supplementary Examinations – March/April 2025

Programme Name: B Tech (ME)

Semester: First

Subject Code: UG11T5102

Subject Name: Engineering Mathematics 1

Date: 04.03.2025	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

Section A **(10X1=10 Marks)**

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. The nullity of an invertible matrix is
   a) 1 b)0 c) 2 d)3

2. If 2ay=x(b+a$\frac{dy}{dx}$), then $y_3 =$
   a) 0 b) constant c) a+b d) None of these

3. Let X and y be two arbitrary, 3x3 nin -zero, skew -symmetric matrices and Z be an arbitrary 3x3 non-zero symmetric matrix, Then which of the following matrices is skew symmetric?
   a) $y^3z^4 - z^4y^3$ b) $x^{44} + y^{44}$ c) $x^4z^3 - z^4x^3$ d) None of these

4) The curve passing through (0,1) and satisfying sin $(\frac{dy}{dx})$ =c′ is

a) cos $\{\frac{(y-1)}{x}\}$=c′ b) sin $\{\frac{(y-1)}{x}\}$=c′ c) cos $\{\frac{x}{(y-1)}\}$=c′

d) sin $\{\frac{x}{(y-1)}\}$=c′

5) The complementary function of $(D^3 + D^2 - D - 1)y = 0$ is

a) $c_1e^x - (c_2 + c_3x)e^{-x}$ b) $c_1e^{-x} + (c_2 + c_3x)e^x$ c) $c_1e^x + (c_2 + c_3x)e^{-x}$

d) $c_1e^{-x} + (c_2 + c_3x)e^x$

6) The number of arbitrary constants in the particular solution of a differential equation of third order is:

a) 3 b) 2 c) 1 d) 0

7)The number of non-zero rows in an echelon form is called?

a) rank of a matrix b) cofactor of the matrix c) reduced echelon form
d) conjugate of the matrix

8) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the

point (-1,-1,2).

a) $\frac{\vec{i}+3\vec{j}+\vec{K}}{\sqrt{11}}$ b) $\frac{\vec{i}-3\vec{j}-\vec{K}}{\sqrt{11}}$ c) $\frac{\vec{i}+3\vec{j}-\vec{K}}{-\sqrt{11}}$ d) $\frac{\vec{i}+3\vec{j}-\vec{K}}{\sqrt{11}}$

9) The value of the line integral $\int_c (2xy^2dx + 2x^2ydy + dz)$ along a path joining the origin and the point (1,1,1) is

a) 0 b) 2 c) 4 d) 6

10) . $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)dxdy =$

a) $\frac{3}{35}$ b) $\frac{7}{60}$ c) $\frac{4}{49}$ d) $\frac{2}{15}$

Section B

Five Questions of 02 Marks each (5X2=10 Marks)

11. If f(x)=$x^3 + 8x^2 + 15x - 24$, calculate the value of $f(\frac{11}{10})$ by the application of Taylor's series.

12. Verify the rank and nullity theorem for the matrix A=$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

13. Find div $\bar{F}$ and curl $\bar{F}$ where $\bar{F}$ = (3x^2 -3yz) i + (3y^2 -3zx) j +(3z^2 -3xy) k
14. Evaluate $\int_0^a \int_0^b \int_0^c x^2y^2z^2 \; dx\, dy\, dz$
15. If u=xy-yz-zx, v=$x^2+y^2+z^2$, $w = x + y - z$, determine whether they are functionally related or not, if so find the relationship between them.

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16.a) Find n[th] derivative of $y = \left[\frac{1}{(x+2)(x+1)(x-1)}\right]$. (05)

b) Solve $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ (05)

17.a) By Euler's theorem for homogeneous function

$u = \frac{(x^2+y^2)^m}{2m\,(2m-1)} + x\varphi\left(\frac{y}{x}\right) + \Psi\left(\frac{y}{x}\right),$

Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)^m$ (05)

17.b) Find the directional derivative of f(x,y,z)=$4e^{2x-y+z}$ at the point (1,1,-1) in the direction towards the point (-3,5,6) (5 marks)

18.a) Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x$ (05)

18.b) Find the maximum, minimum or saddle point of

f(x,y)= $x^3 + y^3 - 3axy$, a>0 (05)

19. a) Find the characteristics equation of the matrix A=$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} . Also, find the matrix represented by

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ (5 marks)

19.b) Prove that $\vec{A} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational. Find the scalar function f(x,y,z) such that $\vec{A} = \nabla f$ (5 marks)

20. a Change the order of integration and hence evaluate the integral

$$\int_0^1 \int_x^{2-x} \frac{x}{y}\,dx\,dy$$ (05)

20. b) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (05)

21. a) Test for consistency $x + y + z = 6$; $x + 2y + 3z = 14$; $x + 4y + 7z = 30$ and solve them. (05)

21.b) Find the Eigen values and Eigen vectors for the matrix A = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

(05)

22. a) Examine whether the given equation is exact or not and hence solve it

$$(x^2 + y^2 + x)dx + xydy = 0$$ (5 marks)

22. b) Find the work done in moving the particle under the field of force given by $\overline{F} = (x^2 - y^2 + x)i - (2xy + y)j$ in the xy plane from the point (0, 0) to (1, 1) along the curve $y^2 = x$. (05)