

Indian Maritime University
(A Central University, Govt of India)

Mar/Apr'26 SE

Programme Name: B Tech (ME)

Semester: First

Subject Code: UG11T5102

Subject Name: Engineering Mathematics 1

Date: 05.03.2026	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
(ii) Options, if any, are specified in respective section.

Section A

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable. 1*10=10marks

1. If $y=x^x$, then $y_2=$

- a) $\frac{1}{y}y_1^2 - \frac{y}{x}$ b) $\frac{1}{y}y_1^2 + \frac{x}{y}$ c) $\frac{x}{y}y_1^2 + \frac{y}{x}$ d) $\frac{1}{y}y_1^2 + \frac{y}{x}$

2. If $z = \cos(xy) + \sin(xy)$, then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$ is equal to:

- a) $-(x^2 + y^2) \sin(xy) + (x^2 + y^2) \cos(xy)$ b) $-(x^2 + y^2) \cos(xy) + (x^2 + y^2) \sin(xy)$
c) 0 d) xy

3. If $x=r \cos \theta$, $y=r \sin \theta$, then $\frac{\partial(r,\theta)}{\partial(x,y)}$ is

- a) r b) 0 c) $\frac{1}{r}$ d) None of these

4. Which integrating factor $f(x)$ depending only on x can make a non-exact

$M dx + N dy = 0$ exact?

- a) $f(x) = e^{\int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} dy}$ b) $f(x) = e^{\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx}$ c) $f(x) = e^{\int M dx}$ d) $f(x) = N(x,y)$

5. If $f(x,y,z)=3x^2y+2y-3z$, then $\nabla f=$

- a) $(6x^2y, 3x^2, 3)$ b) $(6xy, 3x^2 + 2, 3)$ c) $(6xy, 3x^2, -3)$ d) $(6xy, 3x^2 + 2, -3)$

6. For an orthogonal matrix A the valid equality is

- a) $A^T = A^{-1}$ b) $A = A^{-1}$ c) $A^T = A$ d) $\det(A) = 0$

7. If a vector field F is given by $F = (x+2y+mz) i + (2x-3y-z) j + (4x-y+2z) k$ is irrotational, then the value of m is

- a) -3 b) 3 c) -4 d) 4

8. $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dy dx$ is equal to

- a) $e^2 + 1$ b) $e^2 - 1$ c) e^2 d) e^{-2}

9. If A is a square matrix such that $A^2 = A$, then $(1 - A)^3 + A$ is equal to:

- a) 1 b) 0 c) $1 - A$ d) $1 + A$

10. What is the degree of the differential equation $y = x \left(\frac{dy}{dx} \right)^2 + \left(\frac{dx}{dy} \right)$?

- a) 1 b) 2 c) 3 d) 4

Section B

5*2=10 Marks

Five Questions of 02 Marks each

11. Find the nth derivative of $\frac{1}{1+x+x^2}$

12. If $z=f(x+ct)+\Phi(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$

13. Prove that (if $0 < a < b < 1$), $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ using

Lagrange's Mean Value theorem.

14. Solve $(1+y^2) + (x - e^{-\tan^{-1}y}) \frac{dy}{dx} = 0$

15. Using Cayley Hamilton theorem find the inverse of $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.

Section C

5 x 10 = 50 marks

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16a. If $y = e^{a \sin^{-1} x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$

Hence find the value of y_n when $x=0$ (5 marks)

16b. A rectangular box open at the top is said to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.

(5 marks)

17a. If $u = \frac{(x^2+y^2)^m}{2m(2m-1)} + x\Phi\left(\frac{y}{x}\right) + \Psi\left(\frac{y}{x}\right)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)^m$$
 (5 marks)

17b. If $u=f(r)$ and $x=r \cos \theta$, $y=r \sin \theta$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$
 (5 marks)

18a. Prove that $\text{div}(r^n R) = (n+3)r^n$. Hence show that $\frac{R}{r^3}$ is solenoidal.

(5 marks)

18b. A vector field is given by $\vec{A} = (x^2 + xy^2)\vec{i} + (y^2 - x^2y)\vec{j}$. Show that the field irrotational and find its scalar potential.

(5 marks)

19a. Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$.

(5 marks)

19b. Find the complete solution of the differential equation

$$(D^2 + 4D + 3)y(x) = xe^{3x}$$
 (5 marks)

20a. Reduce the following matrix into its normal form and hence find the rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$
 (5 marks)

20b. Solve the following system of linear equations by Gaussian Elimination

Method: $4x+3y+6z=25$, $x+5y+7z=13$, $2x+9y+z=1$ (5 marks)

21a. Change the order of integration in $I = \int_{y=0}^1 \int_{x=y}^1 e^{x^2} dx dy$ and hence

evaluate the same.

(5 marks)

21b. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar co-ordinates and

hence show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

22a. Using green's theorem, evaluate $\int_c [(y - \sin x) dx + \cos x dy]$, where c is

the Plane triangle bounded by the lines $y=0$, $x=\frac{\pi}{2}$ & $y=\frac{2}{\pi} x$.

22b. Find the length of an arc of the cycloid

$x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$.

(5 marks)

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SECTION A

Q No 2

For If $z = \cos(xy) + \sin(xy)$, then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$ is equal to:

- a) $-(x^2 + y^2) \sin(xy) + (x^2 + y^2) \cos(xy)$ b) $-(x^2 + y^2) \cos(xy) + (x^2 + y^2) \sin(xy)$
c) 0 d) xy

Read If $z = \cos(xy) + \sin(xy)$, then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$ is equal to:

- a) $-2xy(\cos(xy) - \sin(xy))$ b) $2xy(\cos(xy) + \sin(xy))$
c) $2xy(\cos(xy) - \sin(xy))$ d) $xy(\cos(xy) - \sin(xy))$

SECTION C

Q No 18 (b)

For - A vector field is given by $\vec{A} = (x^2 + xy^2)\vec{i} + (y^2 - x^2y)\vec{j}$. Show that the field is irrotational and find its scalar potential.

Read - A vector field is given by $\vec{A} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$. Show that the field is irrotational and find its scalar potential.

