

INDIAN MARITIME UNIVERSITY
Time Bound Assignment
B Tech (ME) Arrear Examinations
September/October 2020
UG11T3102
Mathematics-I

Date: 10/09/2020

Duration: 3 Hrs

Max Marks: 70

Pass Marks: 35

- Note:**
- i. Use of approved type of scientific calculator is permitted.
 - ii. The symbols have their usual meanings.

Part – A (compulsory)

Answer the following (10x2=20 Marks)

1. Show that the radius of curvature at any point of the cardioid $r = a(1 - \cos\theta)$ varies as \sqrt{r}
2. Find the nth derivative of $y = \cos x \cos 2x \cos 3x$
3. Using Cayley Hamilton theorem find A^{-1} of matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$
4. Graphically find the minimum value of $Z = 11x + 3y$ subject to the constraints $4x + y \geq 12$, $2x + 2y \leq 18$, $4x - 5y \leq 0$, $x, y \geq 0$
5. Evaluate the integral $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$
6. If $z = e^{ax+by} f(ax - by)$ prove that $\frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$.
7. Find the unit normal vector to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$
8. Find the value of $\Gamma(1/4) \Gamma(3/4)$.
9. If $x = r \cos\theta$, $y = r \sin\theta$ show that $\left[x \frac{\partial x}{\partial r} + y \frac{\partial y}{\partial r} \right]^2 = x^2 + y^2$
10. Find the extremals of the functional $\int_{x_0}^{x_1} \left(\frac{y'^2}{x^3} \right) dx$

Part – B

Answer any 5 out of 7 questions (5 x 10= 50 marks)

- 11(a) If $y = \sin px + \cos px$ prove that $y_n = p^n [1 + (-1)^n \sin 2px]^{1/2}$
(5 marks)
- 11(b) Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$
(5 marks)

12(a) $u = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$ find $\frac{\partial^2 u}{\partial x \partial y}$ **(5 marks)**

12(b) Find the saddle point of $x^3 + y^3 - 3axy$ **(5 marks)**

13(a) Evaluate the double integral $\iint r^3 dr d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$ **(5 marks)**

13(b) Evaluate the triple integral $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2 dx dy dz$ **(5 marks)**

14(a) If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \nabla\phi$ **(5 marks)**

14(b) Find the unit vector tangent to the curve $x = t, y = t^2, z = t^3$ at $t = 1$. **(5 marks)**

15(a) Show that $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is an orthogonal matrix. **(5 marks)**

15(b) If λ is an eigen value of matrix A show that λ^n is eigen value of A^n . **(5 marks)**

16(a) Use the simplex method to solve the LPP $Z = 4x_1 + 3x_2 + 6x_3$ subject to the constraints $2x_1 + 3x_2 + 2x_3 \leq 440$, $4x_1 + 3x_3 \leq 470$, $2x_1 + 5x_2 \leq 430$, $x_1, x_2, x_3 \geq 0$. **(5 marks)**

16(b) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant. **(5 marks)**

17(a) Evaluate $\int_C \frac{z^2 - z + 1}{z - 1} dz$ where C is the circle $|z| = 1$ **(5 marks)**

17(b) Find the curve on which the functional $\int_0^{\pi/2} (y'^2 - y^2 + 2xy) dy$ with $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$, can be extremized. **(5 marks)**

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