Indian Maritime University

(A Central University, Govt of India)

Supplementary Examinations – September/October 2024

Programme Name: B Tech (ME)

Semester: II

Subject Code: UG11T4201

Subject Name: Mathematics II

Date: 13.09.2024 Max Marks: 70

Duration: 03 Hrs Pass Marks: 35

General Instructions

(i) All Sections (A, B & C) are to be attempted.
(ii) Options, if any, are specified in respective section.
(iii) Scientific calculator is permitted.

Section A

Answer All Questions [10x1=10 Marks]

1. The Fourier coefficient b_n for the function

$$f(x) = |x|, \ -\pi < x < \pi$$

A. 1 B. π C. 0 D. $\frac{\pi}{2}$

2. Find the Euler's coefficient a_0 when a function f(x) = x, - π ≤ x ≤ π is expressed as Fourier Series.

A. $-\frac{4(-1)^n}{n\pi}$ B. $\frac{4(-1)^n}{n\pi}$

C. $\frac{2(-1)^n}{n\pi}$ D. 0

3. The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{\frac{1}{3}} = x$ is

A. linear of degree 3 B. non-linear of order 1 and degree 6
C. non-linear of order 1 and degree 2 D. linear of degree 2

4. The orthogonal trajectory of xy = c is

A. $x^2-y^2= c'$ B. $x^2+y^2= c'$ C. $x^2-y^2= 2x$ D. $x^2+y^2= 2y$

5. The complimentary function of $y''-2y'+y=xe^x \sin x$ is

A. $c_1 e^x + c_2 e^{-x}$ B. $(c_1 x + c_2) ex$ C. $(c_1 x + c_2) e^{-x}$ D. $(c_1 + c_2)e^x$

6. The inverse Laplace Transform of $\frac{1}{s^2-9}$ is

a) $\frac{1}{3}\ sinh3t$ b) $\frac{1}{3}\ cosh3t$ c) $\frac{1}{3}\ sin3t$ d) $sinh3t$

7. $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ = _____

A. e^{at} B. e^{-at} C. $t\,e^{-at}$ D. $t\,e^{at}$

8. L(sin t cos t)= _____

9. The value of Cauchy's Integral formula $\oint \frac{3z^2+7z+1}{(z-1)}dz$,where c is the circle $|z| = 1/2$ is

A. $2\pi i$ B. 0 C. πi D. $\frac{\pi i}{2}$

10. For the function $\frac{\sin z}{z^3}$ of a complex variable the point z=0 is

a) a pole of order 4
b) a pole of order 3
c) a pole of order 2
d) not a singularity

Section B

Answer the following: [5x2 = 10 Marks]

11. Find a_0 of the Fourier series expansion for the following function
f(x)=$|x|, -\Pi < x < \Pi$
12. Form the partial differential equation by eliminating the function f from the relation $z = (x + y)f\ (x^2 - y^2)$.
13. Find Laplace Transforms of $(\sin t - \cos t)^2$
14. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$
15. Evaluate using Cauchy's Integral formula $\oint \frac{e^{2z}}{(z+1)^4}dz$,where c is the circle $|z| = 1/2$.

Section C

[5x10 = 50 Marks]

Answer any 5 out of 7 questions.

16. a) Find the Fourier series for
f(x)=x for $0 \le x \le 1$ (05)
= 1-x for $1 \le x \le 2$

b) Find the Fourier series of $f(x)=x^2$ in the interval $(0, 2\pi)$. (05)

17. a) Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$ (05)

b) Solve the given linear differential equation

$2\cos x\frac{dy}{dx} + 4y\sin x = \sin 2x$ (05)

18. a) Solve $(D^2+3D+2)\,y = \sin 2x$ (05)

b) Solve by method of variation of parameters $(D^2-6D+9)y = \frac{e^{3X}}{x^2}$ (05)

19. a) Find the Laplace transform of $L\{\int_0^t e^x x^2\,dx\}$ (05)

b) Evaluate $\int_0^\infty e^{-2t}\left(\frac{e^{at}-\cos bt}{t}\right)$ (05)

20. a) Solve the equation using Laplace Transform

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t, \quad x(0) = 0,\ x'(0) = 1$$ (5 marks)

(05)

b) Using Convolution theorem evaluate $L^{-1}\left[\frac{1}{(s+2)(s+3)}\right]$ (05)

21. a) Evaluate $\oint\frac{(z+3)}{(z+1)(z-2)}\,dz$ where c is the circle $|z| = 3$. (05)

b) Use Cauchy Riemann equation to show that the function

$e^x(\cos y + i\sin y)$ is analytic. Find its derivative. (05)

22. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region

(a) $|z| < 1$ using Taylors series

(b) $1 < |z| < 2$ using Laurent's series (5+5)