

**INDIAN MARITIME UNIVERSITY**  
**Time Bound Assignment September/October 2020**  
**B Sc (Nautical Science) Arrear Examinations**  
**Nautical Mathematics**  
**UG21T4102**

Date: 24/09/2020

Maximum Marks: 70

Time: 3 Hrs

Pass Marks: 35

**Note: Q.No.1 is compulsory.**

**Solve any SIX questions from remaining eight questions.**

**All questions carry equal marks.**

**Use of approved type Scientific Calculator is permitted.**

Q.1 Attempt the following: (10 Marks)

a) Find the nth derivative of  $\log(4x^2 - 1)$ .

b) Find the general value of  $\log(6 + 8i)$ .

c) Compute  $\Gamma\left(-\frac{3}{2}\right)$

d) Evaluate  $\int_0^{\infty} e^{-\sqrt{x}} \sqrt{x} dx$ .

e) In a spherical triangle  $PQR$ , side  $p = 67^\circ$ , side  $q = 54^\circ$ , angle  $P = 39^\circ$ . Find angle  $Q$  using sine formula.

Q.2 a) Evaluate  $\int_0^1 \int_{e^x}^e \frac{dy dx}{\log y}$  by changing the order of integration. (5 Marks)

b) Evaluate  $\iint xy(x+y) dx dy$  over the area bounded by  $y = x^2$  and  $y = x$ . (5 Marks)

Q.3 a) Evaluate  $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z dx dy dz$ . (5 Marks)

b) Prove that  $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m, m)$ . (5 Marks)

Q.4 a) In a spherical triangle  $ABC$ , angle  $A = 88^\circ 36'$ , angle  $B = 121^\circ 36.5'$  and angle  $C = 69^\circ 34.5'$ . Find side  $c$ . (5 Marks)

b) In a spherical triangle  $PXY$ , side  $p = 53^\circ 20'$ , angle  $X = 92^\circ 5'$  and angle  $Y = 90^\circ$ . Find angle  $P$  and side  $y$ . (5 Marks)

Q.5 a) In a spherical triangle  $ABC$ , side  $a = 49^\circ 9'$ , angle  $C = 71^\circ 20'$  and side  $b = 58^\circ 23'$ . Find side  $c$ . (5 Marks)

b) In a spherical triangle  $LMN$ , angle  $N = 81^\circ 50'$ , side  $m = 90^\circ$  and angle  $L = 119^\circ 7'$ . Find side  $n$  and angle  $M$ . (5 Marks)

Q.6 a) If  $y = \sin^{-1}x$ , prove that  $(1 - x^2)y_{n+2} - (2n + 1)x y_{n+1} - n^2 y_n = 0$ . (5 Marks)

b) If  $v = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$  prove that  $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$ . (5 Marks)

Q.7 a) If  $u = F(x - y, y - z, z - x)$  prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$  (5 Marks)

b) Examine the following function for extreme values:

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2 \quad (5 \text{ Marks})$$

Q.8 a) Expand  $\sin^7 \theta \cos^3 \theta$  in a series of sines of multiples of  $\theta$ . (5 Marks)

b) Find  $\tanh x$ , if  $5 \sinh x - \cosh x = 5$ . (5 Marks)

Q.9 a) Prove that  $\log\left(\frac{a+ib}{a-ib}\right) = 2i \tan^{-1}\left(\frac{b}{a}\right)$ . Hence evaluate  $\cos\left[i \log\left(\frac{a+ib}{a-ib}\right)\right]$ . (5 Marks)

b) Find all the values of  $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{\frac{3}{4}}$ . Also show that the continued product of these values is 1. (5 Marks)

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